ANALYSIS OF LOSSY MULTICONDUCTOR TRANSMISSION LINES USING THE ASYMPTOTIC WAVEFORM EVALUATION TECHNIQUE

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Abstract - A new method is described for the transient analysis of lossy coupled transmission line networks with linear or nonlinear terminations. This method is based on an asymptotic waveform evaluation technique which offers two to three orders of magnitude speedup when compared to previously published methods with comparable accuracy. The method is useful for delay and crosstalk simulation of high speed VLSI interconnects.

I. INTRODUCTION

Rapid advances in the development of VLSI circuit technology and packaging techniques are yielding larger chips with smaller and faster devices. As a result, the interconnect delay time is often significantly longer than the device switching time. In addition, as interconnection densities and switching speeds increase, the electrical length of interconnects becomes a significant fraction of a wavelength and conventional lumped-impedance models can no longer be used for accurate simulation of delay and crosstalk. Instead a distributed model for the interconnect should be used in this case[1]-[3].

Several methods have been proposed for the analysis of networks which contain coupled transmission lines [4]-[12]. In most cases, the lossy transmission line has been analyzed either by using a series of lumped models in the time domain, or by using the frequency domain transformation. In general, these techniques provide a detailed analysis of delay and crosstalk, but they require more computer time than the circuit designer can normally afford. The analysis time increases exponentially with circuit size and degree of cross-coupling. This creates the need for a computationally less expensive method which can adequately approximate the circuit response.

In this paper we present a new method for the analysis of lossy coupled transmission line networks with linear or nonlinear terminations. The method has the following advantages:

1. Two or three orders of magnitude speedup compared to previous methods with comparable accuracy,

2. Can handle general transmission line networks with no topological or electrical constraints.

The proposed method is based on Asymptotic Waveform Evaluation (AWE)[13]-[15] in which the transient response is approximated by matching the initial conditions on the first 2q-1 moments of the exact response to a q-pole model.

II. FORMULATION OF THE NETWORK EQUATIONS

Consider a nonlinear network which contains lumped components and arbitrary linear subnetworks. The linear subnetworks may contain distributed components. Without loss of generality the modified nodal admittance matrix equations of the network can be written in the form[16]

$$C\frac{d}{dt}v(t) + Wv(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e(t) - f(v(t)) = 0 \quad (1)$$

where

$v(t) \in \Re^N$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,

$C \in \Re^{N \times N}$ and $W \in \Re^{N \times N}$ are constant matrices with entries determined by the lumped linear components,

$D_k = [d_{i,j}]$, $d_{i,j} \in \{0,1\}$, $i \in \{1,2,...,N\}$, $j \in \{1,2,...,n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $i_k(t) \in \Re^{n_k}$ the vector of currents entering the linear subnetworks k, into the node space $\Re^N$ of the network π,

$e(t) \in \Re^N$ is the vector of independent sources,

$f(v)$: $\Re^N \rightarrow \Re^N$ is a function describing the nonlinear elements of the circuit.

Let the frequency domain equations of the linear subnetwork k be in the form

$$P_k V_k(s) + Q_k I_k(s) = 0 \quad (2)$$

where V_k and I_k represent the Laplace domain terminal voltages and currents of the subnetwork k, respectively.

. In the special case where the subnetwork k consists of a multiconductor transmission line system, P_k and Q_k can be described in terms of the line parameters.

III . Case 1: Lossy Coupled Transmission Lines with Linear Terminations

When the network does not contain any nonlinear elements (1) can be written in the complex frequency domain in the form

$$\begin{bmatrix} sC+W & D_1 & D_2 & \dots & D_{N_s} \\ P_1 D_1^t & Q_1 & 0 & \dots & 0 \\ P_2 D_2^t & 0 & Q_2 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ P_{N_s} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s} \end{bmatrix} \begin{bmatrix} V(s) \\ I_1(s) \\ I_2(s) \\ \cdot \\ \cdot \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} \Theta(s) \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (3)$$

or

$$Y(s) Z(s) = E(s) \quad (4)$$

where

s is the complex frequency

$Z(s) = [V(s) \; I_1(s) \; I_2(s) \dots I_{N_s}(s)]^t$,

$E(s) = [\Theta(s) \; 0 \; 0 \; 0 \dots 0]^t$

$\Theta(s) = L_p[e(t) + D_f x(t)]$; L_p denotes Laplace transform and ^{t} denotes transpose.

To approximate the transient solution z(t) using the asymptotic waveform evaluation technique (4) is expanded in a Maclaurin's series of the form

$$Z(s) = \sum_{n=0}^{\infty} M_n s^n \quad (5)$$

where

$$M_n = \frac{\left.\frac{\partial^n}{\partial s^n}\left[Y^{-1}E\right]\right|_{s=0}}{n!} \quad (6)$$

The moments $m_n=[M_n]_{(i)}$; $n = 0,1,2,\dots,2q-1$ of an output i are then matched to a lower order frequency domain function in the form

$$\left[Z^*(s)\right]_{(i)} = \sum_{j=1}^{q} \frac{k_j}{s-p_j} \quad (7)$$

The approximate time domain transient solution is then

$$\left[z^*(t)\right]_{(i)} = \sum_{j=1}^{q} k_j e^{p_j t} \quad (8)$$

Given the moments $[M_n]_{(i)}$, evaluation of the poles p_j and the residues k_j ; $j=1,2,\dots q$ is described in details in [13]-[14].

Using (4) and (6), a recursive equation for the evaluation of the moments can be obtained in the form

$$[Y_0] M_n = -\sum_{r=1}^{n} \frac{[Y]^{(r)} M_{n-r}}{r!} \quad (9)$$

with

$$[Y_0] M_0 = E \quad (10)$$

Next the derivatives of $\mathbf{Y}$ are obtained as

$$[Y]^{(1)} = \begin{bmatrix} C & 0 & 0 & \dots & 0 \\ P_1^{(1)} D_1^t & Q_1^{(1)} & 0 & \dots & 0 \\ P_2^{(1)} D_2^t & 0 & Q_2^{(1)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ P_{N_s}^{(1)} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s}^{(1)} \end{bmatrix} \quad (11)$$

and

$$[Y]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ P_1^{(r)} D_1^t & Q_1^{(r)} & 0 & \dots & 0 \\ P_2^{(r)} D_2^t & 0 & Q_2^{(r)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ P_{N_s}^{(r)} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s}^{(r)} \end{bmatrix} \; ; (r \ge 2) \quad (12)$$

The derivatives $P_k^{(r)}$ and $Q_k^{(r)}$ can be determined recursively in terms of the line parameters.

IV . Case 2: Lossy Coupled lines with Nonlinear Terminations

In this case the nonzero entries in $f(v(t))$ are replaced by a set of independent waveforms $x(t)$ such that

$$f(v(t)) = D_f x(t) \quad (13)$$

where

$x(t) \in \Re^{N_f}$

$D_f = [d_{ij}]$, $d_{ij} \in \{0,1\}$, $i \in \{1,2,\dots,N\}$, $j \in \{1,2,\dots,N_f\}$ is a selector matrix, and N_f is the number of nonzero entries in $f(v(t))$.

Using (14), (1) is reduced to a set of linear differential equations in the form

$$C\frac{d}{dt}v(t) + Wv(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e(t) - D_f x(t) = 0 \quad (14)$$

The augmenting waveforms $x(t)$ are computed starting with an initial guess $x^0(t)$ and using an iterative technique[17] based on the Newton-Raphson method. For a given set of waveforms $x^{(k)}(t)$, (4) is solved using the AWE technique described in the previous section.

V. Computational Results and Comparisons

Example 1

Consider the circuit shown in Figure 1. Both of the transmission lines are 0.1m long. The two conductor line has the following parameters:

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{nH/m} \qquad \mathbf{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{pF/m}$$

$$\mathbf{R} = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/\text{m} \qquad \mathbf{G} = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \text{S/m}$$

and the parameters of the four conductor line are:

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 & 7.8 & 0.0 \\ 63.3 & 494.6 & 63.3 & 7.8 \\ 7.8 & 63.3 & 494.6 & 63.3 \\ 0.0 & 7.8 & 63.3 & 494.6 \end{bmatrix} \text{nH/m}$$

$$\mathbf{C} = \begin{bmatrix} 62.8 & -4.9 & -0.3 & 0.0 \\ -4.9 & 62.8 & -4.9 & -0.3 \\ -0.3 & -4.9 & 62.8 & -4.9 \\ 0.0 & -0.3 & -4.9 & 62.8 \end{bmatrix} \text{pF/m}$$

$$\mathbf{R} = \begin{bmatrix} 50 & 10 & 1 & 0.0 \\ 10 & 50 & 10 & 1 \\ 1 & 10 & 50 & 10 \\ 0.0 & 1 & 10 & 50 \end{bmatrix} \Omega/\text{m}$$

$$\mathbf{G} = \begin{bmatrix} 0.1 & -0.01 & -0.001 & 0.0 \\ -0.01 & 0.1 & -0.01 & -0.001 \\ -0.001 & -0.01 & 0.1 & -0.01 \\ 0.0 & -0.001 & -0.01 & 0.1 \end{bmatrix} \text{S/m}$$

Figure 2 shows the response at node *b* as calculated using the proposed method, and by numerical inversion of Laplace transformation (NILT) [7]. The applied voltage is a 3ns pulse with 1 nanosecond rise and fall times.

Example 2

To show the efficiency of the proposed technique, we compare is run time with that of HSPICE in table 1. A speed up factor of approximately 45 to 1200, depending on the size of the circuit may be noted. All comparisons were made on a SUN 3/60 workstation. In order to simulate the circuit using HSPICE, lossless transmission lines were assumed. The accuracy of the method is demonstrated in Figures 3 and 4 which show excellent agreement with the results obtained using HSPICE.

Circuit #	Number of TL	Node Count	Lumped Elements	CPU seconds	
				AWE	HSPICE
1	35*	101	145	2.62	256
2	35**	101	145	5.61	261
3	105*	301	435	6.05	7702

*linear terminations
**nonlinear terminations

Table 1. Run time comparision between AWE and HSPICE.

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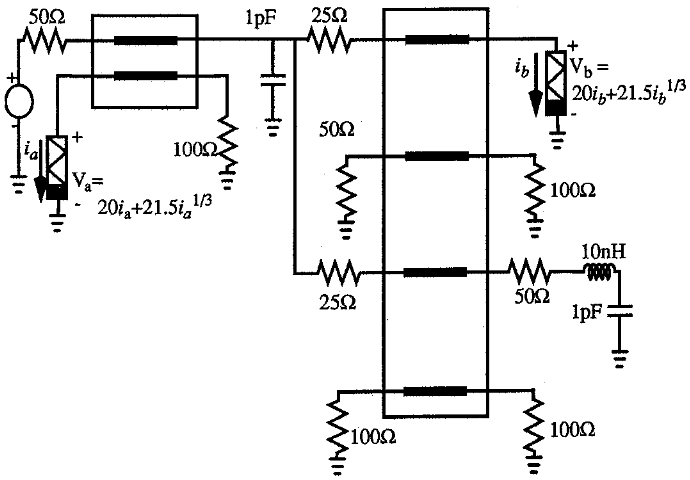

Figure 1. Example Circuit with lossy coupled lines and nonlinear terminations

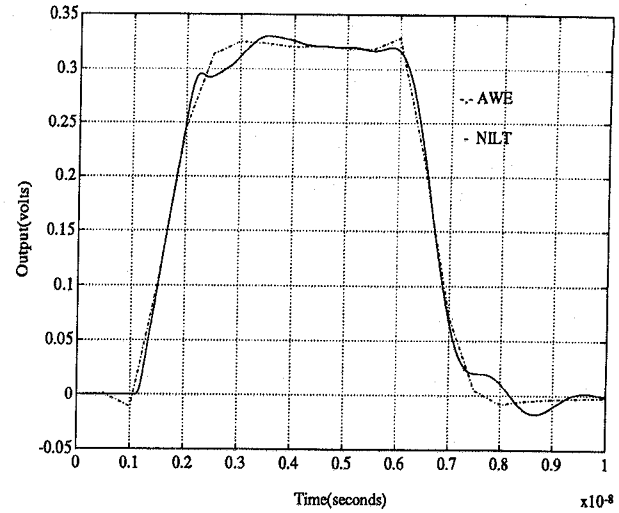

Figure 2. Transient response at node b in the circuit of Fig. 1.

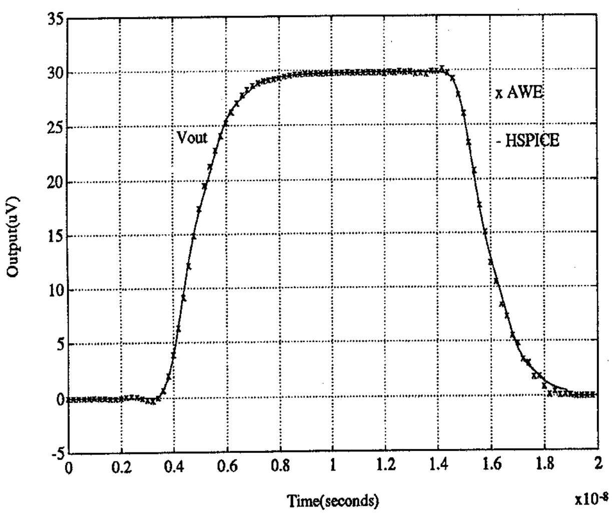

Figure 3. Transient response of circuit #2 in example 2.

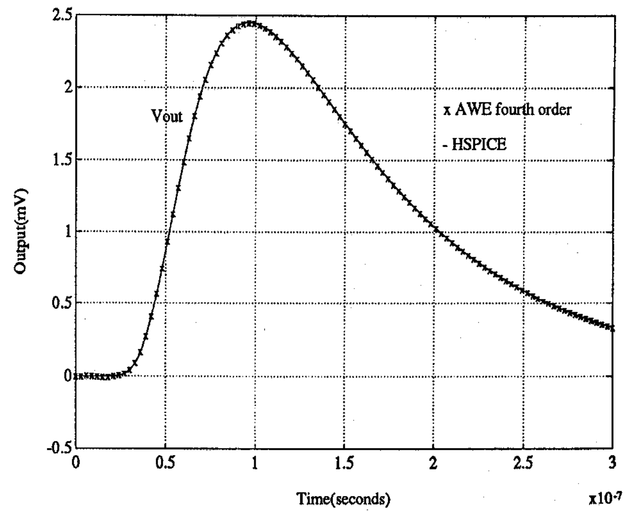

Figure 4. Transient response of circuit #3 in example 2.